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GLUON AND MASSLESS GLUINO SCATTERING USING N=2 SUPERSYMMETRY[1]

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Abstract

The use of N=2 supersymmetry to calculate the matrix elements for the scattering of gluons and massless gluinos, at tree level in perturbative QCD, is discussed. Four, five and six particle processes are used as examples.

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I. INTRODUCTION

Calculations in perturbative QCD quickly become extremely complicated as the number of vertices increases in the Feynman diagrams. This occurs not only for loop calculations but also for tree level calculations. The reasons for this are the increase in the number of complicated three gluon vertices of QCD and the unphysical degrees of freedom which must be removed from the external particles in physical processes. Consider the amplitude for scattering of two gluons into three gluons. At the tree level, there are 25 Feynman diagrams contributing to this process, with each diagram giving $\sim 6^3$ terms. Thus, the amplitude involves thousands of terms which must be squared and then summed over the physical polarizations of the gluons. Such a calculation is only tractable using algebraic manipulation programs.

In this paper, I show how N=2 supersymmetry[1,2] can be used to make such calculations considerably simpler. This is achieved by relating the gluon amplitudes to amplitudes involving scalar particles which have fewer three gluon vertices and, of course, scalar particles have no unphysical degrees of freedom. Using this idea, the matrix element is calculated for many four and five particle processes. In particular, the two gluon to three gluon amplitude is given in only six terms compared to the thousands using standard techiques. Finally, the two gluon to four gluon amplitude is discussed.

II. SUPERSYMMMETRY REVISITED

The particle content of N=2 Yang-Mills theories, consists of a gluon, two species of gluino and a complex scalar. On the mass shell, the gluon and each gluino have two helicities (± 1) and the complex scalar also has two degrees of freedom, which for convenience will be refered to as helicity (± 1). To relate the creation or annihilation operators for these particles, two hermitan operators are formed from the usual generators of N=2 supersymmetry, Q^a_α, as follows

$$Q^a(\eta) = \bar{\eta}^\alpha Q^a_\alpha \tag{1}$$

where η is an arbitrary Majorana spinor. The commutation relations for this $Q^a(\eta)$ with the annihilation operators for the gluon ($g_\pm$), the gluinos ($\lambda^1_\pm, \lambda^2_\pm$) and the scalars ($\phi_\pm$) are

$$
\begin{aligned}
[Q^a, g_\pm] &= \mp\, \Gamma^\pm\ \lambda^a_\pm \\
[Q^a, \lambda^b_\pm] &= \mp\, \Gamma^\mp\ \delta^{ab} g_\pm\ \mp\ i\ \Gamma^\pm \epsilon^{ab}\phi_\pm \\
[Q^a, \phi_\pm] &= \pm\ i\ \Gamma^\mp \epsilon^{ab}\lambda^b_\pm
\end{aligned}
\tag{2}
$$

where

$$\Gamma^+(p,\eta) = (\Gamma^-(p,\eta))^* = \sqrt{2E}(\eta_1 cos\tfrac{1}{2}\theta e^{i\beta/2} + \eta_2 sin\tfrac{1}{2}\theta e^{i\beta/2}) \tag{3}$$

if the representations for the momentum, p, and η are

$$p\ =\ E(\ 1\ ,\ sin\theta cos\beta\ ,\ sin\theta sin\beta\ ,\ cos\theta) \tag{4}$$

$$\eta\ =\ (\eta_1 + \eta_2^*\ ,\ -\eta_1^* + \eta_2\ ,\ -\eta_1 + \eta_2^*\ ,\ -\eta_1^* - \eta_2). \tag{5}$$

Here, η_1 and η_2 are just arbitrary complex numbers.

These commutation relationships can be used to find relationships between different processes in this theory. Consider the scattering of m particles into n particles. If supersymmetry is unbroken, $Q^a(\eta)|vac>= 0$, then the following identity holds for the creation and annihilation operators of these particles,

$$
\begin{aligned}
0 &= <vac|\ [Q^a\ ,\ a_{m+1}\dots a_{m+n}a_1^\dagger\dots a_m^\dagger]\ |vac> \\
&= <vac|\ [Q^a\ ,\ a_{m+1}]a_{m+2}\dots a_{m+n}a_1^\dagger\dots a_m^\dagger\ |vac>\ +\ \dots \\
&\quad +\ <vac|\ a_{m+1}\dots a_{m+n}[Q^a\ ,\ a_1^\dagger]a_2^\dagger\dots a_m^\dagger\ |vac>\ +\ \dots
\end{aligned}
\tag{6}
$$

This equation contains relationships between various processes which are exact for N=2 Yang-Mills theories but only hold to tree level in QCD. One can see this by comparing the Feynman diagrams for both theories at tree level or by appealing to R symmetry.

III. TWO TO TWO PROCESSES

The first process I consider is the scattering of two gluons into two gluons. The commutator of Q^a with $g_{4-}g_{3-}g_{2+}^\dagger\lambda_{1+}^\dagger$ leads to the following equation

$$
\begin{aligned}
&\Gamma_1^+ <|\ g_{4-}g_{3-}g_{2+}^\dagger g_{1+}^\dagger\ |>\ +\ \Gamma_2^- <|\ g_{4-}g_{3-}\lambda_{2+}^\dagger\lambda_{1+}^\dagger\ |> \\
&+\ \Gamma_3^- <|\ g_{4-}\lambda_{3-}g_{2+}^\dagger\lambda_{1+}^\dagger\ |>\ +\ \Gamma_4^- <|\ \lambda_{4-}g_{3-}g_{2+}^\dagger\lambda_{1+}^\dagger\ |>\ =\ 0
\end{aligned}
\tag{7}
$$

where $\Gamma_i^{\mp} = \Gamma^{\pm}(p_i, \eta)$. Remember, Γ^+ depends on η_1, η_2 and Γ^- on η_1^*, η_2^* and since each of these is arbitrary, eqn(7) is really four equations. One of these is simply

$$< | \; g_{4-} g_{3-} g_{2+}^{\dagger} g_{1+}^{\dagger} \; | > \; = \; 0. \tag{8}$$

Similar manipulations also lead to

$$< | \; g_{4-} g_{3+} g_{2+}^{\dagger} g_{1+}^{\dagger} \; | > \; = \; 0. \tag{9}$$

Finally, all the nonzero helicity amplitudes for this process can be obtained by crossings from

$$\begin{aligned} \mathcal{M}(g_{1+}, g_{2+}; g_{3+}, g_{4+}) &= e^{i\theta_{1234}} \mathcal{M}(\lambda_{1+}, \lambda_{2+}; \lambda_{3+}, \lambda_{4+}) \\ &= e^{i\psi} \mathcal{M}(\phi_{1+}, \phi_{2+}; \phi_{3+}, \phi_{4+}). \end{aligned} \tag{10}$$

The phase factor $e^{i\theta_{1234}}$ accounts for the different spin statistics properties of the gluons and gluinos whereas $e^{i\psi}$ is an overall phase factor and is irrelevant.

This result is now used to calculate the cross section for gluon gluon scattering using the N=2 Yang-Mills Lagrangian,

$$\begin{aligned} \mathcal{L} &= -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} - (D_\mu \phi)^{a\dagger} (D^\mu \phi)^a \\ &\quad -\frac{1}{2} g^2 (f^{abc} \phi^b \phi^{c\dagger})(f^{ade} \phi^{d\dagger} \phi^e) + fermion\ terms. \end{aligned} \tag{11}$$

Scalar scalar scattering in this theory is very simple and it is easily shown that

$$\mathcal{M}(\phi_{1+}, \phi_{2+}; \phi_{3+}, \phi_{4+}) \; = \; -2ig^2 \; f_{x13} \; f_{x24} \; \frac{(1 \cdot 2)}{(1 \cdot 3)} \; + \; (3 \Leftrightarrow 4) \tag{12}$$

where $(i \cdot j) \; = \; (p_i \cdot p_j)$. After squaring, and suitably summing and averaging over helicity and color, the well known result is obtained

$$|\mathcal{M}_{gg \to gg}|^2 \; = \; \frac{4g^4 N^2}{(N^2 - 1)} \; [3 - st/u^2 - tu/s^2 - us/t^2] \tag{13}$$

where s, t and u are the Mandelstam variables.

The next process to consider is gluino gluino production by gluon gluon fusion. Taking the commutator of Q^a with $g_{4+} g_{3+} g_{2+}^{\dagger} \lambda_{1+}^{\dagger}$, gives

$$\begin{aligned} &\Gamma_1^+ < | \; g_{4+} g_{3+} g_{2+}^{\dagger} g_{1+}^{\dagger} \; | > \; + \; \Gamma_2^- < | \; g_{4+} g_{3+} \lambda_{2+}^{\dagger} \lambda_{1+}^{\dagger} \; | > \\ &+ \; \Gamma_3^+ < | \; g_{4+} \lambda_{3+} g_{2+}^{\dagger} \lambda_{1+}^{\dagger} \; | > \; - \; \Gamma_4^+ < | \; \lambda_{4+} g_{3+} g_{2+}^{\dagger} \lambda_{1+}^{\dagger} \; | > \; = \; 0. \end{aligned}$$

The Γ^+ terms give two equations in three unknowns, hence

$$|\mathcal{M}(\lambda_{1+}, g_{2+}; \lambda_{3+}, g_{4+})| = \sqrt{\frac{(1\cdot 4)}{(3\cdot 4)}}\, |\mathcal{M}(g_{1+}, g_{2+}; g_{3+}, g_{4+})|. \tag{15}$$

Again, after squaring and suitably summing and avervaging over helicity and color, gives the following

$$|\mathcal{M}_{gg\to\lambda\lambda}|^2 = \frac{g^4 N^2}{(N^2-1)} \left[tu(t^2+u^2)(s^2+t^2+u^2)/s^2t^2u^2\right]. \tag{16}$$

This agrees with DEQ[3] for a massless gluino.

IV. TWO TO THREE PROCESSES

For the scattering of two gluons into three gluons, all helicity amplitudes are crossings of $\mathcal{M}(g_{1+}, g_{2+}; g_{3\pm}, g_{4+}, g_{5+})$ or are zero to tree level. Once again supersymmetry can be used to relate this amplitude to an amplitude involving scalars,

$$|\mathcal{M}(g_{1+}, g_{2+}; g_{3\pm}, g_{4+}, g_{5+})| = (1\cdot 2)^{\pm 1}(4\cdot 5)^{\mp 1}\, |\mathcal{M}(\phi_{1+}, \phi_{2+}; g_{3\pm}, \phi_{4+}, \phi_{5+})|. \tag{17}$$

The evaluation of the scalar amplitude is quite simple if one uses the polarization tensors, $\epsilon_\mu(p_3, p_m)$, of Xu, Zhang and Chang,[4] where

$$\epsilon_\mu p_n^\mu = \pm \frac{<m:n><3:n>}{\sqrt{2}<m:3>} \tag{18}$$

where p_3 is the gluon momentum , p_m is the reference momentum and $<m:n>$ is the spinor inner product of the two momenta p_m and p_n. The only property of this product which is of interest to us here is

$$<m:n>^*<m:n> = 2(m\cdot n). \tag{19}$$

The result for the scalar matrix element is

$$\begin{aligned}
\mathcal{M}(\phi_{1+}, \phi_{2+}; g_{3+}, \phi_{4+}, \phi_{5+}) \sim& \\
& f_{x1y}\, f_{3y4}\, f_{x25}\, \frac{<1:2><4:5>}{<1:3><3:4><2:5>} \\
&+ f_{x2y}\, f_{3y5}\, f_{y14}\, \frac{<1:2><4:5>}{<2:3><3:5><1:4>} \\
&+ f_{x14}\, f_{y25}\, f_{3yx}\, \frac{<1:2>^2<4:5>}{<1:3><2:3><1:4><2:5>} \\
&+ (4 \Leftrightarrow 5)
\end{aligned} \tag{20}$$

and eqn(17) can be used to get the purely gluonic matrix element. Note the simplicity of this expression! There are six independent color factors for this process and six terms in the final result. The linear independence of the color factors implies that each term is gauge independent and therefore each term can be evaluated with a different (convenient) reference momentum.

The third and sixth terms in this expression are the only complications, a part from the color factors, caused by the non-abelian nature of the gluon. The rest is just scalar QED. Also, all denominators are, up to a phase factors, square roots of poles. Therefore the required cancellation of all double poles[5] for the purely gluonic matrix element squared has been achieved at the matrix element level!

The square of this expression is easy obtained and after summing over color indices gives

$$|\mathcal{M}(g_{1+},g_{2+};g_{3+},g_{4+},g_{5+})|^2 \sim (1\cdot 2)^4 \sum_P \frac{1}{(1\cdot 2)(2\cdot 3)(3\cdot 4)(4\cdot 5)(5\cdot 1)} \qquad (21)$$

where $\sum_P$ is the sum over all permutations of $1 \ \ldots \ 5$. After the appropriate sum and average of helicity and color, the final expression is

$$|\mathcal{M}_{gg\to ggg}| = \frac{-g^6 N^3}{240(N^2-1)}[\sum_P (1\cdot 2)^4] \sum_P \frac{1}{(1\cdot 2)(2\cdot 3)(3\cdot 4)(4\cdot 5)(5\cdot 1)}. \qquad (22)$$

Again the power of this method can be demonstrated by relating this result to that of the process of two gluons scattering into two massless gluinos and a gluon. The appropriate relationships can be obtained as before,

$$|\mathcal{M}(g_{1+},g_{2+};\lambda_{3-},g_{4+},\lambda_{5+})| = \sqrt{\frac{(3\cdot 4)}{(4\cdot 5)}}\,|\mathcal{M}(g_{1+},g_{2+};g_{3-},g_{4+},g_{5+})| \qquad (23)$$

$$|\mathcal{M}(g_{1+},g_{2-};\lambda_{3+},\lambda_{4-},g_{5+})| = \sqrt{\frac{(2\cdot 3)}{(2\cdot 4)}}\,|\mathcal{M}(g_{1+},g_{2-};g_{3+},g_{4-},g_{5+})|. \qquad (24)$$

The final result for this process is

$$|\mathcal{M}_{gg\to g\lambda\lambda}|^2 = \frac{-g^6 N^3}{20(N^2-1)} [\,[\,(1\cdot 4)^3(1\cdot 5) + (1 \Leftrightarrow 2) + (1 \Leftrightarrow 3)\,] + (4 \Leftrightarrow 5)\,]$$
$$\sum_P \frac{1}{(1\cdot 2)(2\cdot 3)(3\cdot 4)(4\cdot 5)(5\cdot 1)}. \qquad (25)$$

An interested reader can now enjoy calculating $|\mathcal{M}_{\lambda\lambda\to g\lambda\lambda}|^2$ in a similar manner.

V. TWO TO FOUR PROCESSES

In this section I consider the two gluon to four gluon process. Here I define all helicities as if all particles are incoming. Once again supersymmetry can be used to show that to tree level

$$|\mathcal{M}(g_{1-},g_{2-},g_{3-},g_{4-},g_{5-},g_{6-})| = 0, \tag{26}$$

$$|\mathcal{M}(g_{1-},g_{2-},g_{3-},g_{4-},g_{5-},g_{6+})| = 0, \tag{27}$$

$$|\mathcal{M}(g_{1-},g_{2-},g_{3-},g_{4-},g_{5+},g_{6+})| = \frac{s_{56}}{s_{23}}\,|\mathcal{M}(g_{1-},\phi_{2-},\phi_{3-},g_{4-},\phi_{5+},\phi_{6+})|, \tag{28}$$

and

$$\begin{aligned}|\mathcal{M}(g_{1-},g_{2-},g_{3-},g_{4+},g_{5+},g_{6+})| = \frac{1}{s_{23}s_{56}}|&(s_{12}+s_{23}+s_{13})^2\mathcal{M}(g_{1-},\phi_{2-},\phi_{3-},g_{4+},\phi_{5+},\phi_{6+})\\ &-2i\sqrt{-s_{14}}(s_{12}+s_{23}+s_{13})(p_5^x+p_6^x-ip_5^y-ip_6^y)\mathcal{M}(\lambda_{1-},\phi_{2-},\phi_{3-},\lambda_{4+},\phi_{5+},\phi_{6+})\\ &-s_{14}(p_5^x+p_6^x-ip_5^y-ip_6^y)^2\mathcal{M}(\phi_{1-},\phi_{2-},\phi_{3-},\phi_{4+},\phi_{5+},\phi_{6+})\ |\end{aligned} \tag{29}$$

where this last equation is given in the center of mass frame of particles 1 and 4. Unfortunately, a compact analytical result for this process has not yet been obtained but fast numerical programs[6] have been written which allow both experimentialists and theorists to study this process. It is worth noting that various checks can be applied to these programs especially permutation symmetry and the absence of double poles when two momenta are made parallel.

Methods similar to that used in the two to three gluonic process might lead to a simple expression for the amplitude which can then be squared without producing an enormous number of terms.

VI. DISCUSSIONS

The use of supersymmetry is a powerful tool in reducing the complicated calculations required when dealing with gluons in perturbative QCD. There are many areas one could extent the basic ideas of this paper. Can one include quarks into this scheme? Yes.[2,7] Are higher order calculations simplified by using the supersymmetry relations and subtracting out the extra graphs not found in ordinary QCD? What happens if you include masses which do not break supersymmetry? If they do break supersymmetry? How about extenting these ideas to the complete standard model.

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